where, without loss of generality, we have set $\Omega_1 = 0$. When $D_1 = D_2 = \Omega_2 = 0$, we recover the results of Abrams et al. [1]. In particular, $r_1 = 1$ (θ_i^1 all equal) is invariant. If r_1

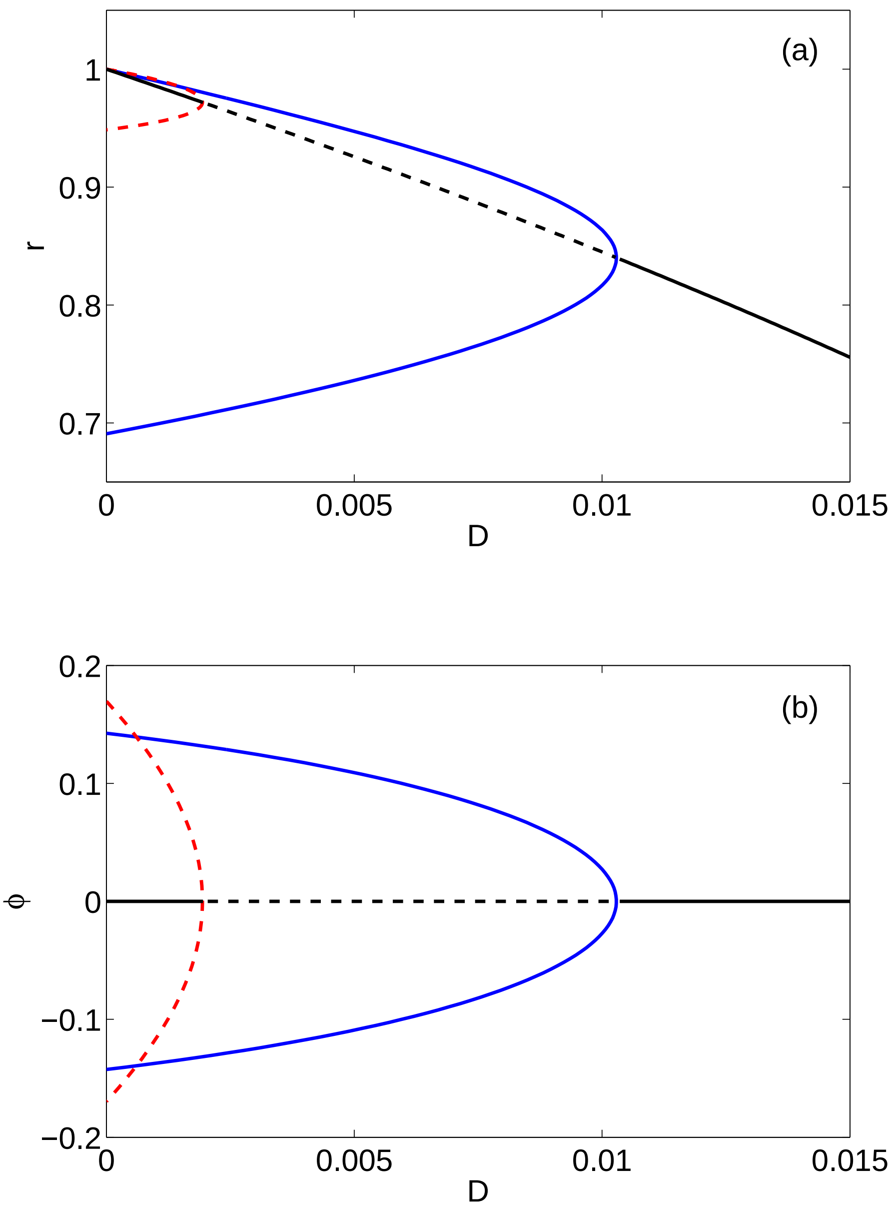
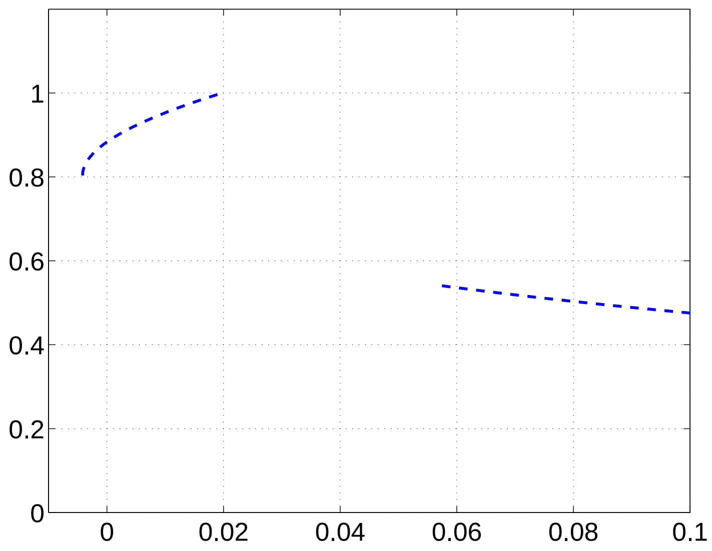


Figure 1: (Colour online) Fixed points of (11)-(13) when $D_1 = D_2 = D, \Omega_2 = 0$. (a): r_1 and r_2 as a function of D. (b): φ as a function of D. Blue curve: stable chimera. Red curve: saddle chimera. Black curve: symmetric state ($r_1 = r_2$). Solid lines indicate stable solutions, dashed lines unstable. Other parameters: $A = 0.2, \beta = 0.07$.



1
0.8
0.6
0.4
0.2
0
0
0.02
0.04
0.06
0.08
0.1

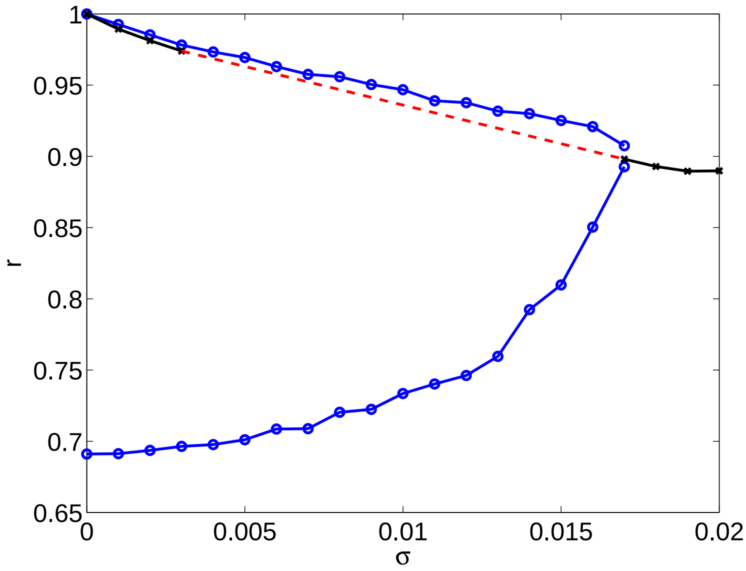


Figure 4: (Colour online) r_1 and r_2 fitted to simulations of (1), where all ω_i^k are chosen from a normal distribution of mean zero and standard deviation σ. Blue circles joined by a line: stable chimera. Black crosses joined by a line: stable symmetric state ($r_1 = r_2$). Red dashed line: presumed unstable symmetric state. Compare with Fig. 1. See the text for details on the fitting. Other parameters: $A = 0.2, \beta = 0.07, N = 1000$.

and found chimera states, both for homogeneous and heterogeneous networks.

4 Other distributions

Now we consider the effects of choosing the ω_i^k from distributions other than the Lorentzian, first numerically and then analytically.

4.1 Gaussian distribution: numerical simulations

Figure 4 shows the results of fitting the time-dependent PDF

$$
\begin{aligned}
f_k(\theta, t) &= \frac{1}{2\pi}\left\{1 + \left[\sum_{n=1}^{\infty}\left(r_k e^{i\phi_k}\right)^n e^{in\theta} + \text{c.c.}\right]\right\} \\
&= \frac{1 - r_k^2}{2\pi[1 - 2r_k\cos(\phi_k - \theta) + r_k^2]} \qquad (14)
\end{aligned}
$$

to each population in simulations of (1) after transients, where all ω_i^k are chosen from a normal distribution of mean zero and standard deviation σ. We found that both r_1 and r_2 tended to constant values, as did $\phi_2 - \phi_1$ (not shown). Only stable states are shown in Fig. 4, but the results are compatible with those shown in Fig. 1, suggesting that there is nothing special about the Lorentzian distribution, as has been noted by others [14, 7]. The unstable states could presumably be found using the "equation-free" method [13, 8, 12] of analysing low-dimensional descriptions of high-dimensional systems, under the assumption that these states are also exactly described by the variables r, ϕ

6 Oscillators on a ring

In some cases this double integral can be exactly evaluated. We follow [3] and suppose that $G(x) = (1 + A\cos x)/(2\pi)$, so that $G(x - y) = (1 + A\cos x\cos y + A\sin x\sin y)/(2\pi)$. Let us define

$$h(y) = \int_{-\infty}^{\infty} \left(\frac{\omega - \Omega - \sqrt{(\omega - \Omega)^2 - R^2(y)}}{R(y)} \right) g(\omega)d\omega$$

Thus under the assumption that R and Θ are even (which can be shown to be self-consistent)

$$R(x)e^{i\Theta(x)} = c + a\cos x \tag{22}$$

where

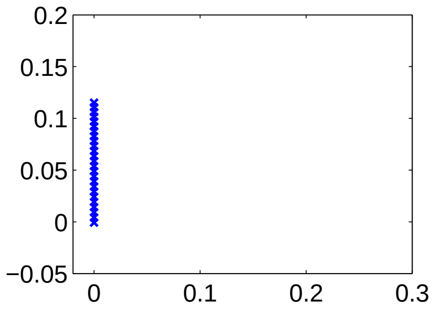
$$c = \frac{e^{i\beta}}{2\pi} \int_0^{2\pi} e^{i\Theta(y)} h(y)dy \tag{23}$$

and

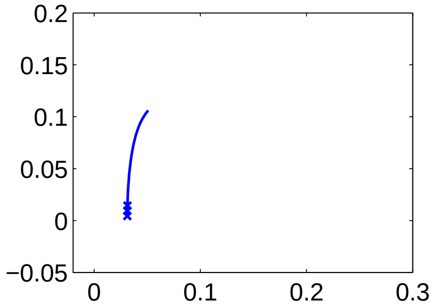
$$a = \frac{Ae^{i\beta}}{2\pi} \int_0^{2\pi} e^{i\Theta(y)} h(y)\cos y\, dy \tag{24}$$

Since (21) is unchanged by the shift Θ15(s)-3.04115]TJΩR3498D49.0TdΩ[TfΩ4.319840TdΩ[(x)5.40839]TJ

π



0.2
0.15
0.1
0.05
0
−0.05
0
0.1
0.2
0.3



0.2
0.15
0.1
0.05
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−0.05
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0.1
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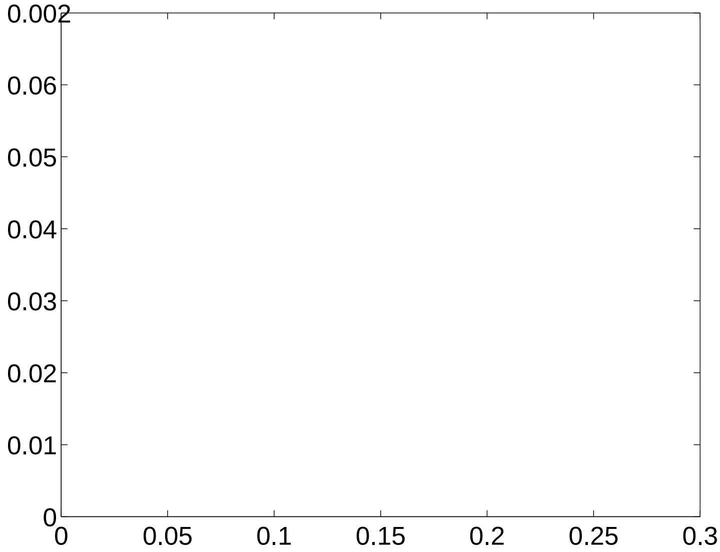
0.3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0.06
0.05
0.04
0.03
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0.01
0
0
0.05
0.1
0.15
0.2
0.25
0.3

6.4 Chimera states and "bumps"

Chimera states as studied in this section are very similar to "bump" states which have been studied in computational neuroscience modelling

(2008), 084103.

[2] Abrams, D. M., and Strogatz, S. H. Chimera states for coupled oscillators. Phys. Rev. Lett. 93 (2004), 174102.

[3] Abrams, D. M., and Strogatz, S. H. Chimera states in rings of nonlocally coupled oscillators. Int. J. Bifn. Chaos 16 (2006), 21–37.

[4] Acebrón, J., Bonilla, L., Pérez Vicente, C., Ritort, F., and Spigler, R.

[19] Pikovsky, A., Rosenblum, M., and Kurths, J. Synchronization. Cambridge University Press, 2001.

[20] Ren, L., and Ermentrout, B. Phase locking in chains of multiple-coupled oscillators. Physica D 143, 1-4 (2000), 56–73.

[21] Sethia, G., Sen, A., and Atay, F. Clustered Chimera States in Delay-Coupled Oscillator Systems. Phys. Rev. Lett. 100 (2008), 144102.

[22] Singer, W. Neuronal Synchrony: A Versatile Code for the Definition of Relations? Neuron 24 (1999), 49–65.

[23] Strogatz, S. From Kuramoto to Crawford: exploring the onset of synchronization in populations of coupled oscillators. Physica D 143 (2000), 1–20.

[24] Strogatz, S. Sync: The Emerging Science of Spontaneous Order. Hyperion, 2003.

[25] Watanabe, S., and Strogatz, S. Integrability of a globally coupled oscillator array. Physical Review Letters 70 (1993), 2391–2394.

[26] Watanabe, S., and Strogatz, S. Constants of motion for superconducting Josephson arrays. Physica. D 74 (1994), 197–253.

[27] Wiesenfeld, K., Colet, P., and Strogatz, S. Synchronization transitions in a disordered josephson series array. Phys. Rev. Lett. 76 (1996), 404–407.