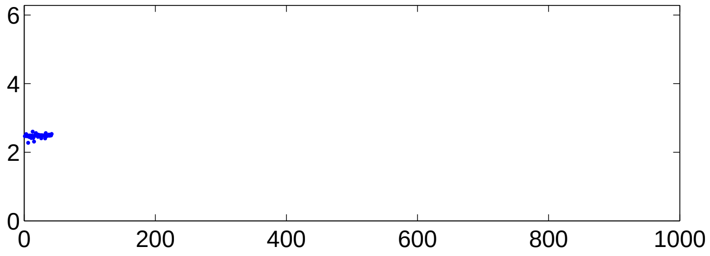
system, in the limit of an infinite number of oscillators, could be described by three ordinary differential equations. However, Pikovsky and Rosenblum [20] showed, using the ansatz of Watanabe and Strogatz [21], that the Ott/Antonsen (OA) ansatz did not completely d


6
4
2
0
0
200
400
600
800
1000

A, B, C and **D**), but where each member of the ensemble has a randomly chosen set of frequencies $\{\omega_j^1\}$ and $\{\omega_j^2\}$ (these both come from the same distribution **g**, and **g** is the same for each member of the ensemble). Letting the number of members of the ensemble go to infinity we describe the state of population 1 by the probability density function

$$f^1(\theta_1^1, \theta_2^1, \ldots, \theta_N^1; \omega_1^1, \omega_2^1, \ldots \omega_N^1; t)$$

and population 2 by the function

$$f^2(\theta_1^2, \theta_2^2, \ldots, \theta_N^2; \omega_1^2, \omega_2^2, \ldots \omega_N^2; t)$$

which, by conservation of oscillators [4, 18], satisfy

$$\frac{\partial f^\sigma}{\partial t} + \sum_{j=1}^{N} \frac{\partial}{\partial \theta_j^\sigma}\left(f^\sigma \left(\frac{d\theta_j^\sigma}{dt}\right)\right) = 0 \tag{7}$$

for **σ**

and

$$\bar{A}_j = \frac{1}{N}\sum_{k=1}^{N} A_{jk}\bar{a}_k \qquad \bar{B}_j = \frac{1}{N}\sum_{k=1}^{N} B_{jk}\bar{b}_k \tag{27}$$

$$\bar{C}_j = \frac{1}{N}\sum_{k=1}^{N} C_{jk}\bar{b}_k \qquad \bar{D}_j = \frac{1}{N}\sum_{k=1}^{N} D_{jk}\bar{a}_k \tag{28}$$

The interpretation of the a_k is that the magnitude of a_k gives the "peaked-ness" of the angular distribution over the ensemble of the θ_k^1 — the closer $|a_k|$ is to 1 the more peaked the distribution, while $|a_k| = 0$ corresponds to a uniform angular distribution. The argument of a_k gives the phase about which the distribution of the θ_k^1 are peaked. Similarly for the b_k and population 2.

Note that when A, B, C and D are full, there exists a solution of (23)-(28) for which $a_k = a$ and $b_k = b\ \forall k$, where a and b are governed by the two complex equations studied by Laing [3]:

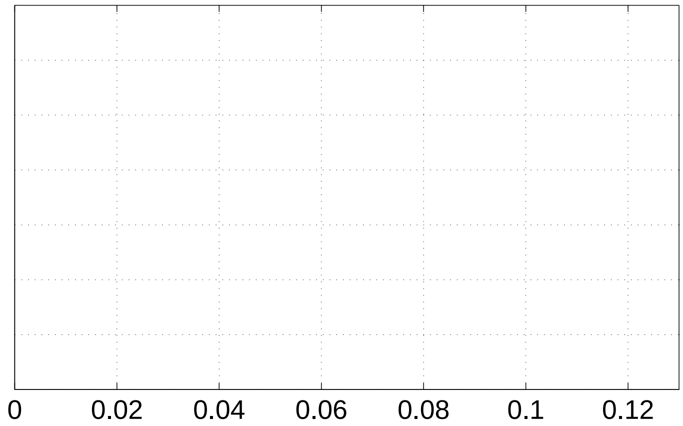
$$\frac{da}{dt} = -\Delta a + (e^{i\alpha}/2)(\mu a + \nu b) - (e^{-i\alpha}/2)(\mu\bar{a} + \nu\bar{b})a^2 \tag{29}$$

$$\frac{db}{dt} = -\Delta b + (e^{i\alpha}/2)(\mu b + \nu a) - (e^{-i\alpha}/2)(\mu\bar{b} + \nu\bar{a})b^2 \tag{30}$$

If $\Delta = 0$, (29)-(30) are the same equations as studied by Abrams et al. [7]. As Barlev et al. [8] noted, this type of sychronised solution, for which $a_k = a$ and $b_k = b\ \forall k$, also exists when the coupling matrices all have the same row sum, i.e. all of the oscillators have the same in–degree. Note that (29)-(30) were derived by Abrams et al. [7] not by averaging over an infinite ensemble of finite networks as done by Barlev et al. [8], but by considering the limit $N \to \infty$ for a single network with full connectivity.

III. RESULTS

We now consider the solutions of (23)-(28). For concreteness we set $\Delta = 0.001$; we also define $\beta = \pi/2-$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0.02
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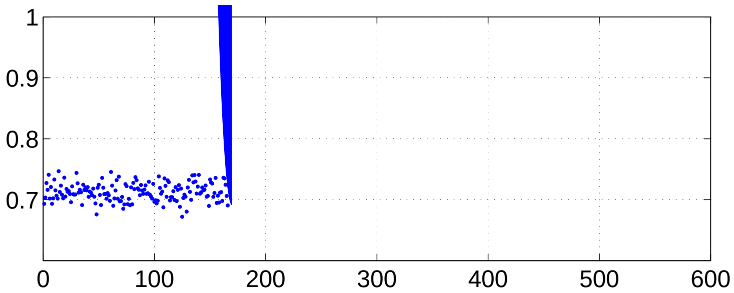
0.06
0.08
0.1
0.12


1
0.9
0.8
0.7
0
100
200
300
400
500
600

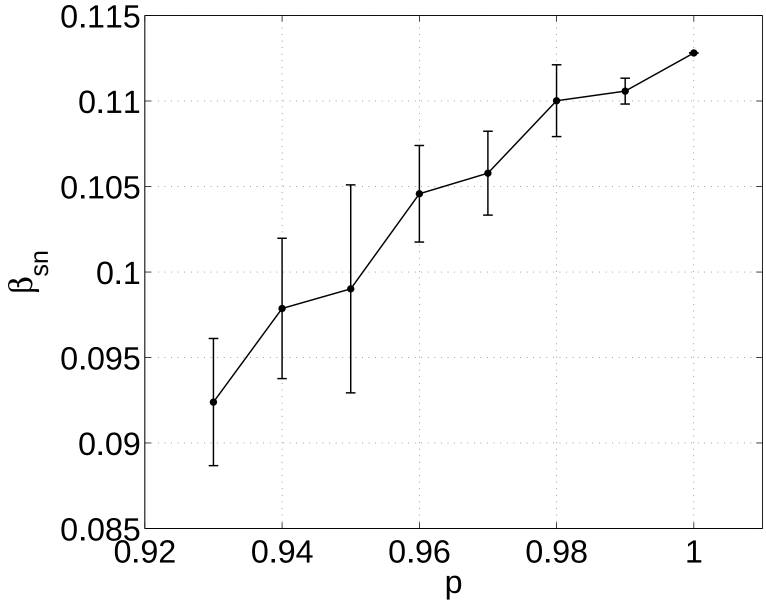


FIG. 7: The values of β_{sn} for which there is a saddle-node bifurcation of fixed points of (25)-(28) and (31)-(32) for Erdös-Rényi-type random matrices A, B, C and D, as a function of p. At each value of p, 10 realisations of the random matrices were used, and the mean and standard deviation of the 10 β_{sn} are shown. Other parameters: $N = 300, E = 0.2, \Delta = 0.001$.

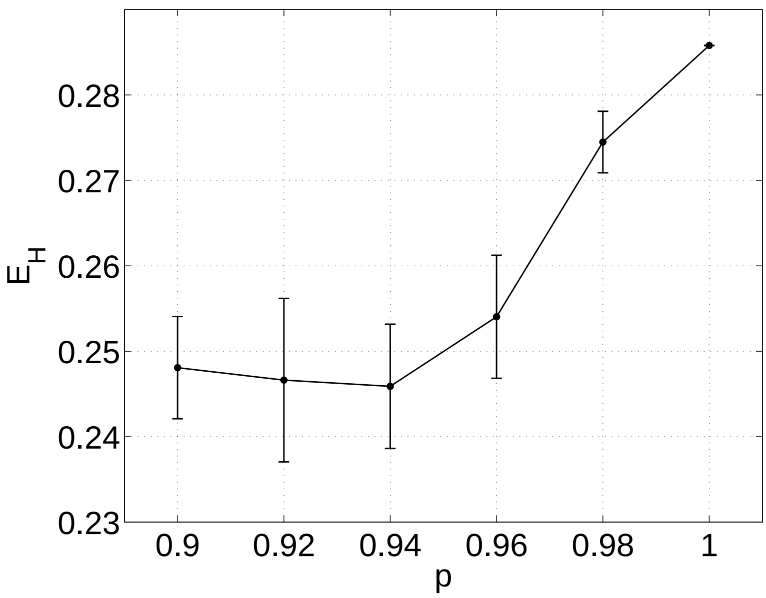
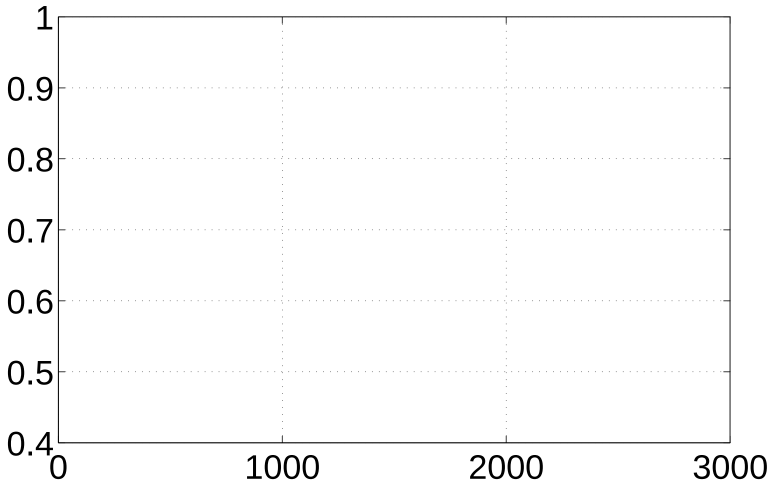


FIG. 8: Values of E at which a Hopf bifurcation of a stable stationary chimera state occurs for an Erdös-Rényi-type network, for different values of p. At each value of p, 10 realisations of the random matrices were used, and the mean and standard deviation of the 10 values of E are shown. Other parameters: $N = 300, \beta = 0.05, \Delta = 0.001$.

2. Chung-Lu-type networks

We now consider a second type of perturbation from the fully-connected case in which edges are removed preferentially so as to create a specific skewed degree distribution. The algorithm we use to create these networks is motivated by the Chung-Lu algorithm [26]. We begin by assigning to each oscillator in a subnetwork a weight $w_i = N(i/N)^r$, where $i = 1, 2, \ldots N$ is the oscillator number within the network.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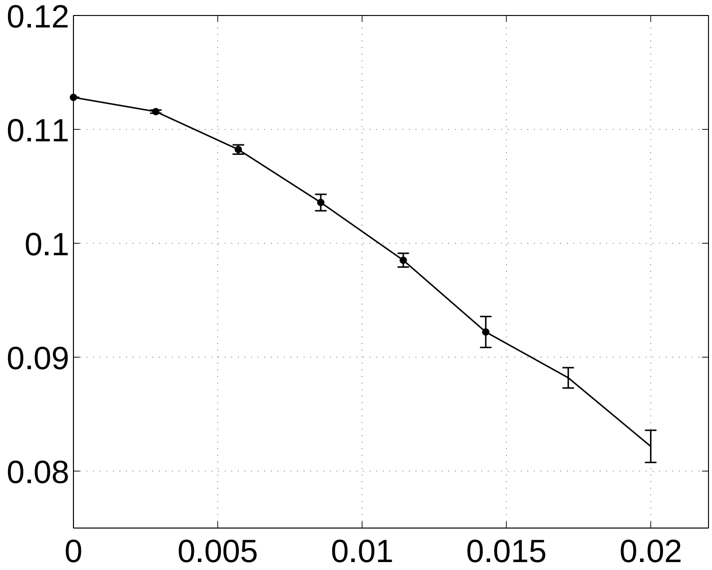
0.8
0.7
0.6
0.5
0.4
0
1000
2000
3000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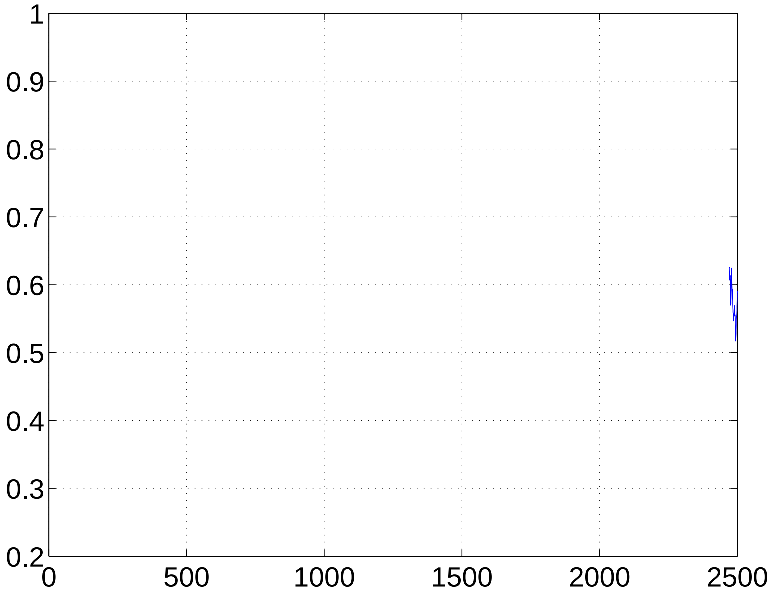
0.11
0.1
0.09
0.08
0
0.005
0.01
0.015
0.02


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0
500
1000
1500
2000
2500

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