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Successive homoclinic tangencies to a limit cycle

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Abstract

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The dynamics near a perturbed degenerate homoclinic connection to a periodic orbit in three dimensions is modeled by

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4. Simple fixed points

The simplest periodic orbits of the flow are the trajectories which link up with themselves after only one pass through the global region of the flow. These correspond to fixed points $(x, y) \in \Sigma_a$ of our mapping which satisfy $(x, y) = L^m G(x, y)$, or more explicitly

$$x = \lambda_s^m \phi(y) ,$$

$$y = \lambda_u^m \left\{ \left[\mu + \gamma \bar{x}(x, y) \right] y / y_0 + \epsilon f(y) \right\} .$$
(20)

The substitution of the first equation $x = \lambda_s^m \phi(y)$ into the second equation window equation for a descended fixed points merge again in a second saddle-node tangency and disappear. Thus as μ varies from positive to negative, two cycles are created in a saddle-node bifurcation and then the same two cycles merge and are destroyed in another saddle-node bifurcation.

Now we find where the saddle-node bifurcations occur relative to the primary homoclinic tangencies. The saddle nodes occur when

$$\boldsymbol{\beta} = -\boldsymbol{\epsilon} f'(\boldsymbol{y}) \,, \tag{24}$$

and a simple computation shows that the corresponding fixed point $(\lambda_{x}^{m}\phi(v), v)$ is also given by a solu-

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of a flow and our model map, a plot of orbit period versus a parameter path through the two tangencies yields closed bubbles.

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other example where two tangency curves come together in parameter space and the associated saddlenode bifurcations at the separate tangencies may be linked together. Period versus parameter plots in this situation also show the accumulation of saddle-node bifurcations on homoclinic tangencies, but in this case the orbits are linked by a single curve which zigzags in Kirk and Edgar Knobloch for useful comments and conversations regarding this paper.

References

 S.N. Chow, J.K. Hale and J. Mallet-Paret, An example of hifurcation to homoclinic orbits_J. Diff. Eans. 37 (1983)