PHASE OSCILLATOR NETWORK MODELS OF BRAIN DYNAMICS

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Abstract. Networks of periodically firing neurons can be modelled as networks of coupled phase oscillators, each oscillator being described by a single angular variable. Networks of two types of neural phase oscillators are analysed here: the theta neuron and the Winfree oscillator. By taking the limit of an infinite number of neurons and using the Ott/Antonsen ansatz, we derive and then numerically analyse "neural field" type diﬀerential equations which govern the evolution of macroscopic order parameter-like quantities. The mathematical framework presented here allows one eﬃciently simulate such networks, and to investigate the eﬀects of changing the structure of a network of neurons, or the parameters of such networks.

1. Introduction

It is well established that a single neuron can fire a periodic train of action potent o

ones. The study of oscillations in neuroscience is a large topic [55, 56, 20, 10, 54] and

identical. Thus we can determine the asymptotic dynamics of (6) by assuming that F is given by (9). It is helpful to introduce the complex order parameter, as considered by Kuramoto in the context of coupled phase oscillators [30, 53]

$$z(t) \equiv \int_{-\infty}^{\infty} \int_{0}^{2\pi} F(I, \theta, t) e^{i\theta} \, d\theta \, dI. \tag{10}$$

The quantity z can be thought of as the expected value of $e^{i\theta}$. Substituting the ansatz (9) of the infinite network. This pair of equations was studied with $\tau = 0$, i.e. instantaneous synapses, by [39]. For a physical interpretation of $z \in \mathbb{C}$, write $z(t) = r(t)e^{i\psi(t)}$. Integrating (9) over I we obtain the probability density function

$$p(\theta, t) = \frac{1 - r^2(t)}{2\pi\{1 - 2r(t)\cos[\theta - \psi(t)] + r^2(t)\}} \tag{18}$$

which is a unimodal function of θ with maximum at $\theta = \psi$, and whose sharpness is governed by the value of r [35, 34]. Alternatively, we follow [42] and define

$$w \equiv \frac{1 - \bar{z}}{1 + \bar{z}} = \frac{1 + 2ir\sin\psi - r^2}{1 + 2r\cos\psi + r^2}. \tag{19}$$

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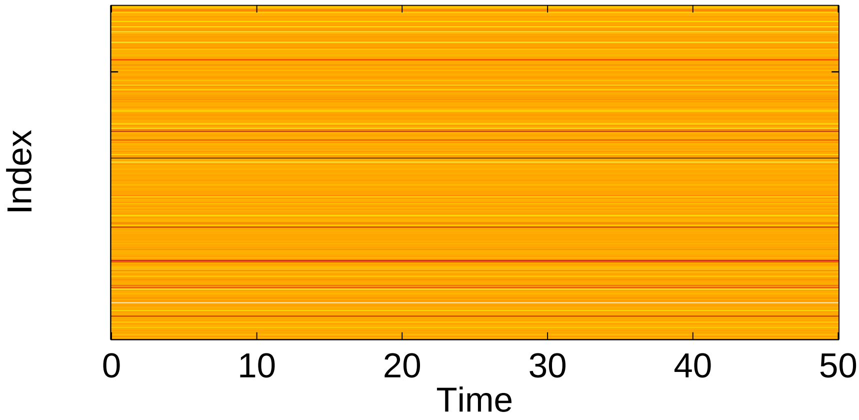
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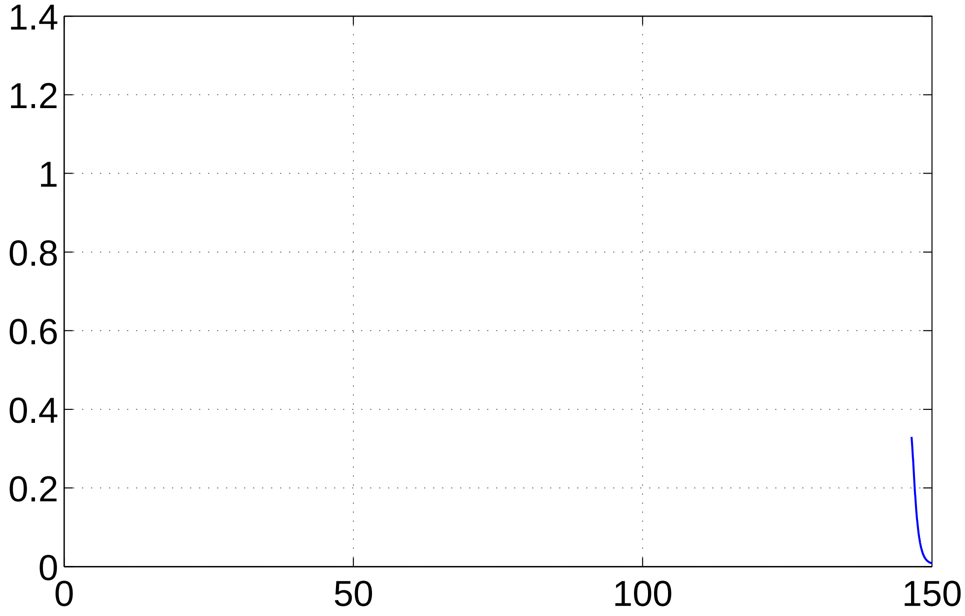
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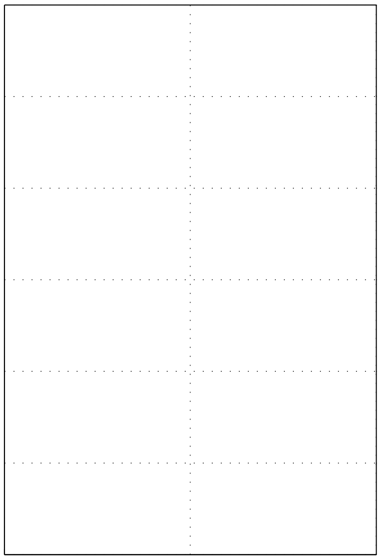

and have

$$z(t) \equiv \int_{-\infty}^{\infty} \int_{0}^{2\pi} F(\omega, \theta, t) e^{i\theta} \, d\theta \, d\omega. \tag{42}$$

equations are then

$$\frac{\partial z(x,t)}{\partial t} = \frac{\square R(x,t)e^{-i\beta}}{2\gamma} + (i\omega$$

References

[23] Boris Gutkin. Theta-neuron model. In Dieter Jaeger and Ranu Jung, editors,