Delays in activity based neural networks

By Stephen Coombes

provide a caricature of the behaviour of more realistic spiking networks when the time-scale of synaptic processing is much longer than the membrane time constant of a typical cell (Ermentrout 1986). This is perhaps most clearly demonstrated by recent work of Roxin et al. (2005), which further emphasises that a single delay in the activity based representation can further improve the match with spiking networks. The delay in the activity based model is interpreted by them as describing the time course of AP initiation. However, an alternative interpretation of this delay is that it is necessary to adequately model the time-lag involved in generating a rate based representation of a spiking network. In particular single neuron firing rates (for slow synapses) will be largely determined by the steady state values of non-spiking currents, and thus the delay may be more naturally interpreted as the time for these currents to relax. In any case, this paper will show how to analyse a delayed neural network with a hybrid approach, combining linear stability theory, the construction of periodic orbits (for piece-wise constant nonlinear firing rate functions) and numerical techniques.

2. The model

As discussed above, under certain approximations spiking network models can be reduced to just a few variables. One famous example is the Wilson & Cowan (1972) model, which describes the evolution of a network of synaptically interacting neuronal populations (typically one being excitatory and the other inhibitory). In the presence of delays this model takes the form

$$\begin{aligned} \dot{u} &= -u + f(\theta_u + au(t-\tau_1) + bv(t-\tau_2)), \\ \frac{1}{\alpha}\dot{v} &= -v + f(\theta_v + cu(t-\tau_2) + dv(t-\tau_1)). \end{aligned} \tag{2.1}$$

Here, u and v represent the synaptic activity of the two populations, with a relative time-scale for response set by α^{-1}. The architecture of the network is fixed by the weights a, b, c, d, whilst $\theta_{u,v}$ describe background drives (biases). The firing rate function f is commonly chosen as a sigmoid:

$$f(z) = \frac{1}{1+e^{-\beta z}}, \tag{2.2}$$

Hopf and saddle-node bifurcations. The point (u^*, v^*) is an equilibrium if there is a solution to the pair of equations

$$\theta_u = f^{-1}(u^*) - au^* - bv^*, \qquad \theta_v = f^{-1}(v^*) - cu^* - dv^*, \tag{2.3}$$

where $f^{-1}(z) = \beta^{-1} \ln(z/(1-z))$. The Jacobian matrix is therefore

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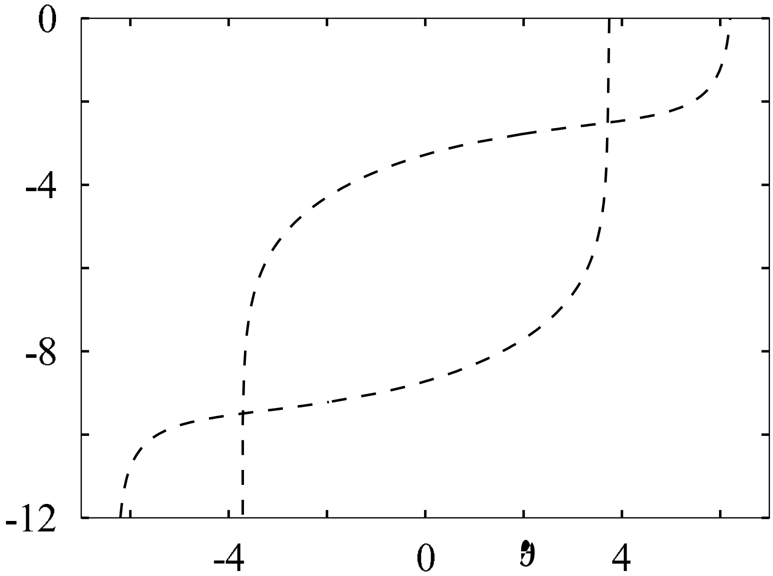

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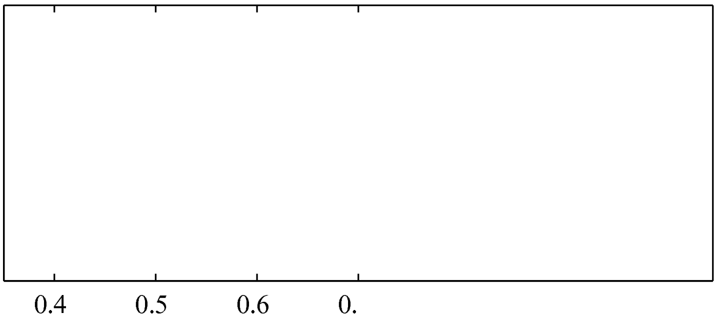
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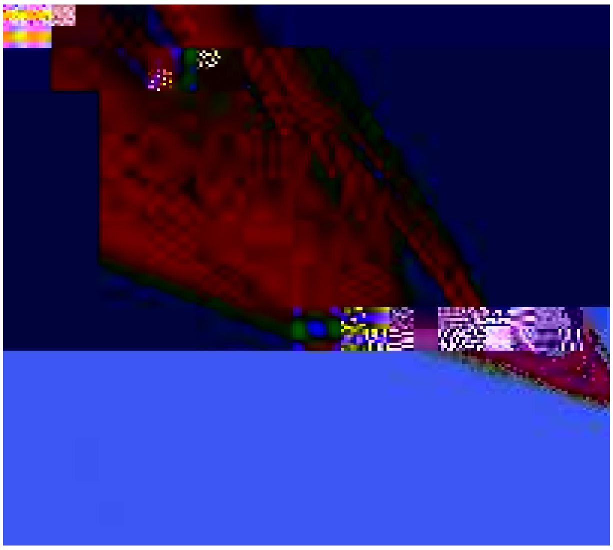

the origins of bursting in low dimensional ODEs is quite well understood there has been very little work on bursting in delay differential equations. Here we briefly

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