# Chimeras in networks of planar oscillators

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Chimera states occur in networks of coupled oscillators, and are characterized by having some fraction of the oscillators perfectly synchronized, while the remainder are desynchronized. Most chimera states have been observed in networks of phase oscillators with coupling via a sinusoidal function of phase di erences, and it is only for such networks that any analysis has been performed. Here we present the first analysis of chimera states in a network of planar oscillators, each of which is described by both an amplitude and a phase. We find that as the attractivity of the underlying periodic orbit is reduced chimeras are destroyed in saddle-node bifurcations, and supercritical Hopf and homoclinic bifurcations of chimeras also occur.

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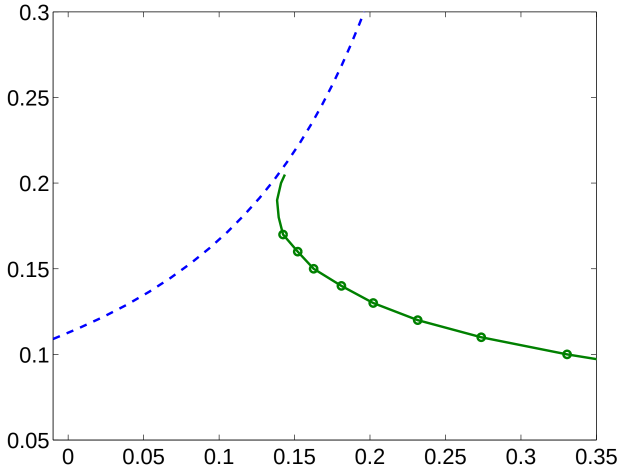
Networks of coupled oscillators and their synchronization properties have been studied for many years [1, 2]. One particular class of interest involves phase oscillators, and Eq. (2) can be written as a similar pair of equations. From Eq. (3) we see that as $\Box \to 0$, the rate of attraction to the limit cycle $r_j = 1\ \forall j$ becomes infinite, and Eq. (4) reduces to Equation (1) of [11] (after a redefinition of $\omega$), i.e. our system reduces to a previously-studied network of phase oscillators. We will investigate the dynamics of (1)-(2) when $\Box \neq 0$. By allowing the radius $r$ to vary, we expect a wider variety of behaviour than that seen in networks of phase oscillators; for example, oscillator death and chaos [30]. For comparison with previous results we define $\beta = \pi/2 - \alpha$ and we let $\mu = (1+A)/2, \nu = (1-A)/2$, where $A$ is a parameter [11].

Firstly, we show a chimera state for (1)-(2); see Fig. 1. Panel (a) shows a snapshot of all $\theta_j$ at an arbitrary time. We see that population two (with $N + 1$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